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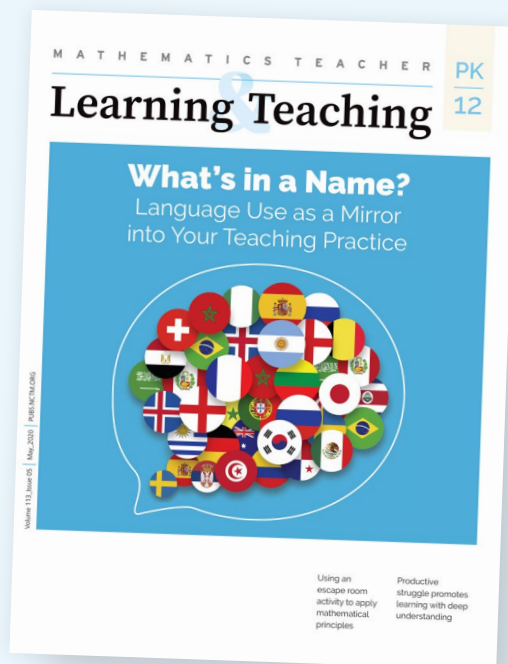
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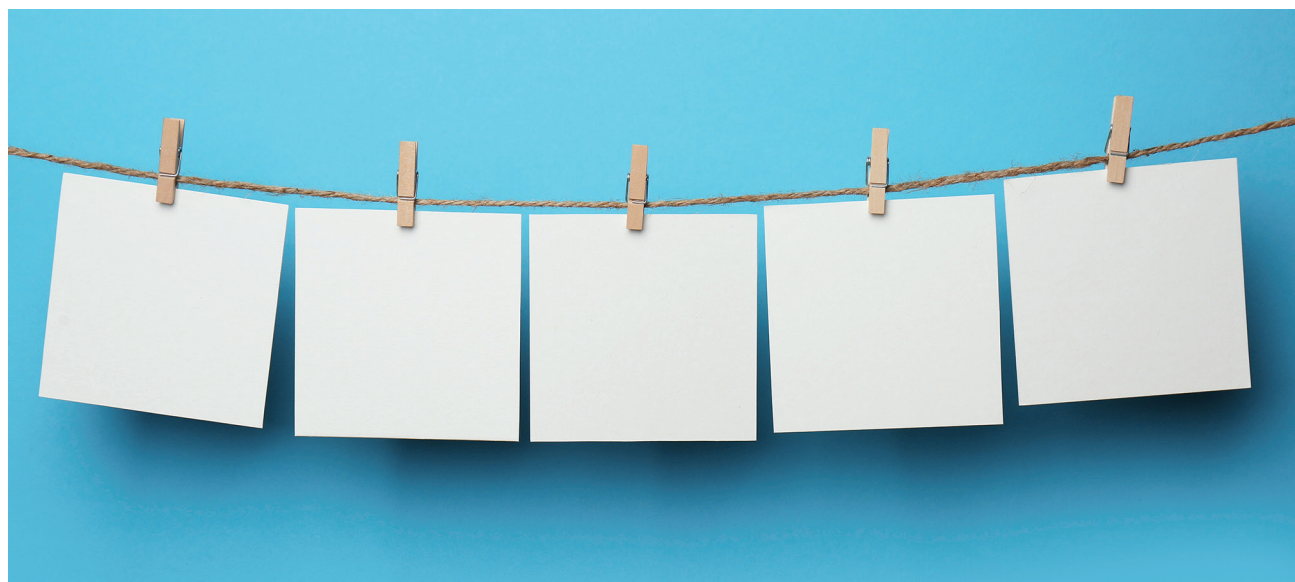
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Engaging Early Childhood Students Using Clothesline Math

Strengthen young learners' agency and accountability by problematizing number sense, place value, and multiplication concepts using a Clothesline Math routine.

Stacy K. Boote, Terrie M. Galanti, Danielle Felicien, and Tara Kelly

Classroom routines provide predictability while protecting instructional time as students engage in “small cooperative scripts of behavior” (Leinhardt et al., 1987, p. 1). While many routines support the general goal of classroom management, well-designed mathematics routines serve another important purpose. They provide opportunities for students to reflect and discuss their sense making while exploring

mathematical relationships. Mathematics routines can “open up conversations and thinking about mathematics that might not happen by themselves” (McCallum & Nowak, 2018, as cited in Berry, 2018, slide 9).

Teachers and teacher educators have been sharing strategies and resources for implementing mathematics routines in National Council of Teachers

of Mathematics (NCTM) journals for years. As early childhood mathematics teacher educators and classroom teachers, we empower students using “Notice and Wonder” (Fetter, 2024), “Contemplate then Calculate” (Lucenta & Kelemankik, 2022), and “Numberless Word Problems” (Mahaffey, 2021). These routines elevate the quality of learning environments by fostering student-centered discussions of mathematical relationships. As students discuss, model, and debate their sense making, they make their mathematical reasoning visible and explicit.

USING A CLOTHESLINE MATH ROUTINE WITH YOUNG LEARNERS

A less commonly shared mathematics routine, especially with young learners, is “Clothesline Math” (Shore, 2017, 2018). In this routine, teachers create student opportunities to integrate spatial reasoning with other concepts by sequencing mathematical representations on life-size number paths (Marcella-Burdett et al., 2020). Students hang number tents made from index cards or printed from Internet sources (see supplemental material [link online]) on one or more clotheslines (see Video 1).

The yarn on which tents are placed can be physically held or attached to surfaces using magnets, tape, or glue across classroom spaces (e.g., tables, walls, bulletin boards, carpet). Students create tangible number paths on the classroom clothesline (Figure 1). Number representations on tents are

created using word forms, expanded notations, tick marks, dot patterns, tens-frames, Roman numerals, clocks, and base-ten blocks (Figure 2). Table 1 provides recommendations for planning, implementing, and reflecting on Clothesline Math.

Clothesline Math allows multiple entry points as Pre-K-3 students move along developmental trajectories for counting, subitizing, comparing numbers, and adding/subtracting (Clements & Sarama, 2017/2019). As students move tents showing mixed groupings of quantity (Figure 3; Acosta, 2021), they are relying on more than memorized counting strategies. Consequently, students have opportunities to subitize *conceptually* instead of just *perceptually* (Sarama & Clements, 2009) to support the

Video 1 Clothesline Math Implementation Ideas



 Watch the full video online.

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construction of arithmetic units (Wilkins et al., 2022). Students can also begin to mentally rearrange quantities using emerging understandings of place value concepts (Chan et al., 2014).

Using tangible number paths and physical interactions with mathematical concepts are asset-based approaches to supporting children (Suh et al., 2021) as they progress through thinking levels within developmental trajectories. We use “number path” to provide a contrast with number lines. A number *path* supports ordinality and spacing conversations without eliciting more developmentally advanced conversations about scale or equal intervals on a number *line*. While Clothesline Math increases student engagement and makes exploring mathematical relationships enjoyable and interactive, it also creates

equitable participation opportunities for productive conversations. Moreover, engaging in challenging problem solving using multimodal representations (Celedon-Pattichis et al., 2017) helps students become community members where all ideas of number and counting are valued and heard (Franke et al., 2023).

CLOTHESLINE MATH AS AN APPROACH TO PRODUCTIVE DISCIPLINARY ENGAGEMENT

Engle and Conant (2002) offered a *productive disciplinary engagement* model to characterize equitable mathematics learning environments where students are genuinely participating in classroom discourse. Four guiding principles of productive disciplinary

Figure 1 Clothesline Math Routine in a Pre-K Classroom

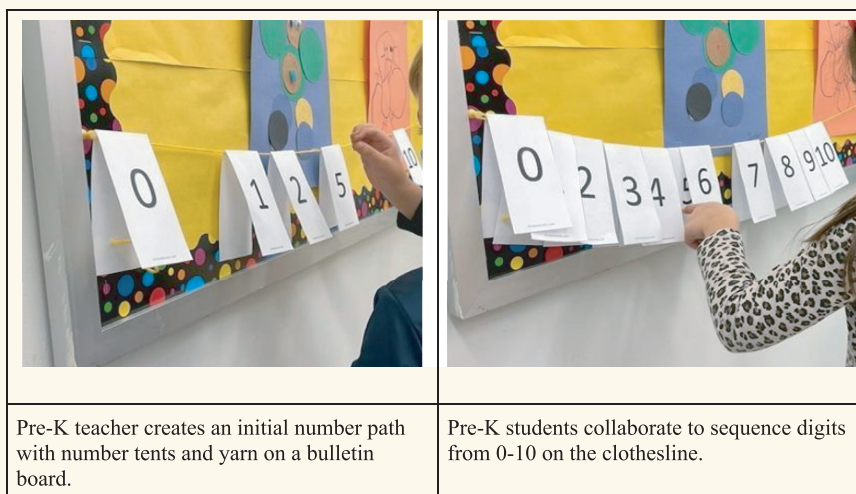
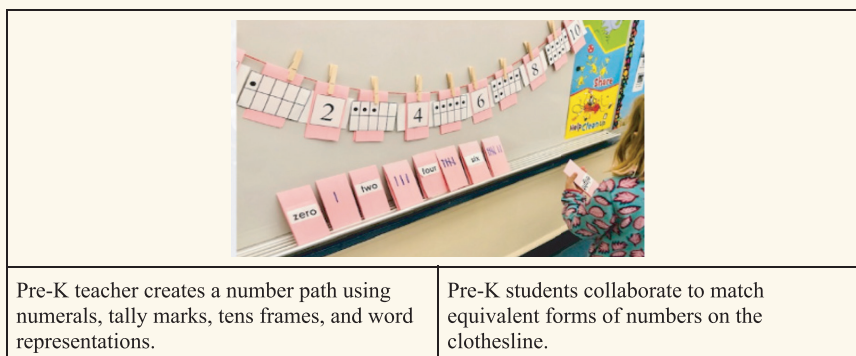


Figure 2 Clothesline Math Routine in a Pre-K Classroom



engagement (Engle, 2012) adapted for mathematics classrooms are as follows: 1) **problematizing mathematics** beyond recall of facts and procedures; 2) **giving students authority** to develop and share

mathematical ideas; 3) **holding students accountable** to themselves and others for equitable participation; and 4) **providing access to necessary resources** to promote problematizing, authority, and accountability.

Table 1 Enacting the Clothesline Math Routine for Early Childhood Students

Steps	Recommendations
Planning and Preparing	<p>Create tents aligning with lesson/unit objective(s) using index cards folded in half. Tents can be printed from Internet sources (see supplemental material [link online] for Internet sources).</p> <p>Place the clothesline at a student-friendly height (e.g., bulletin board, smart board, edge of table, floor) where students can sequence tents.</p> <p>Place benchmark numbers (e.g., 0, 5, 10) on the clothesline before starting to provide access and support.</p>
Implementing	<p>Hand out tents and ask students to sequence them in numerical order on the clothesline. Teachers may call on students individually or in groups.</p> <p>Invite students to look over the initial placement of tents and notice and wonder about tents they would place in different locations.</p> <p>If a student wants to move a tent, ask the student who initially placed the tent to explain their reasoning. Encourage students to discuss suggested movements.</p>
Reflecting	<p>Teachers can leave up the clothesline with tents as resources for future discussions and problem-solving tasks.</p>

Figure 3 Sample Clothesline Math Number Tent Images for Early Counting and Number Concepts



Note: Adapted from Acosta (2021)

Table 2 describes how these principles can support teachers as they create Clothesline Math routines so all students are expected and empowered to share their mathematical thinking.

While Clothesline Math can be used in any grade, we describe implementations by two early childhood teachers enrolled in an online mathematics and technology course facilitated by university mathematics educators serving as Co-PI's on Project InTERSECT (link online), a US Department of Education STEM professional development SEED grant. Each week, teachers learned a new mathematics routine in a Canvas learning management system module with hyperlinks to free resources before designing their own to implement with students. They shared evidence of implementations (e.g., text, photos, and videos) with colleagues using collaborative Google Slideshow templates. Tamara, a Pre-K teacher, used Clothesline Math to facilitate counting and identifying numbers 1–20. Diane, a Grade 3 teacher, used Clothesline Math to review multi-digit addition and skip counting

patterns and build students' understanding of multiplication concepts.

Tamara's Pre-K Clothesline Math Routine

Tamara implemented Clothesline Math over multiple days, allowing students to explore subitizing and counting ordinal numbers 1–20. Tamara *problematized* subitizing and sequencing by providing tents showing numeral and dot pattern representations. Her use of multiple representations as *resources* created mathematical challenges because students were held *accountable* for subitizing within a counting task. Students were also given *authority* to reason and struggle when they felt there would not be space for their tent based on other students' tent placements.

Day 1. Tamara invited students to sit in front of their class clothesline on the whiteboard. She gave students authority to participate in problem solving by handing each person a tent. Some had pattern dots, while others had numerals.

Table 2 Examples of Using Productive Disciplinary Engagement Principles to Guide Clothesline Math Routine Creations in Pre-K–3 Classrooms

Productive Disciplinary Engagement Principles in Mathematics Classrooms (Adapted from Engle, 2012)	What is the teacher doing in a Clothesline Math Routine?	What are students doing in a Clothesline Math Routine?
<p>Problematizing Students are encouraged to take on mathematical challenges, ask questions, and collaborate with peers to resolve uncertainties.</p> <p>Authority Students play an active role in identifying, formulating, and resolving problems as they discuss, model, and debate their mathematical sensemaking.</p> <p>Accountability Students express their ideas and listen to others as they work together to make their mathematical sensemaking visible.</p> <p>Resources Teacher provides sufficient resources for students to engage in mathematical problem solving.</p>	<p>Teacher poses a mathematical challenge to students related to counting and sequencing.</p> <p>Teacher invites student(s) to place their tent on the clothesline.</p> <p>Teacher elicits student ideas about placements and potential adjustments.</p> <p>Teacher encourages student(s) to look for patterns and use spatial reasoning to justify their decisions.</p> <p>Teacher attends to equitable participation in facilitating the negotiations about number placements.</p> <p>Teacher asks probing questions to navigate mistake making and disagreements.</p> <p>Teacher makes connections to prior mathematical discussions or learning goals for upcoming lessons.</p>	<p>Students think about where to place their tent as they wait their turn.</p> <p>Students think about where to place their tent on the clothesline as they watch other students make choices about number path positions.</p> <p>Students think about changes they would make to tent placements.</p> <p>Students wait to express their ideas until all the tents have been placed.</p> <p>Students offer suggestions for moving tents and justify their reasoning.</p> <p>Students respectfully negotiate when they disagree with the placement of a tent.</p>

Tamara held all students accountable by calling them to place their tents on the clothesline. After five students placed tents in the correct sequence, Myra struggled to place her tent, having a dot pattern for 6, within the spacing of tents with numeral representations (Figure 4).

Although Myra could “singsong” 1–20 number counts, she was confused about where to place a representation of the 6 tent using two columns of three dots within a sequence of numerals. Myra clapped to celebrate her success after placing her tent to the far right of the numeral 20 tent. She believed her tent needed its own place because it did not look like a numeral.

Myra’s contribution created a new mathematical problem. Tamara called on Chris, whose hand was raised. “I have a ‘gestion (suggestion),” he said as Tamara asked Chris to join Myra at the clothesline to share his idea (Figure 5). Chris picked the dot card tent representing 6 and moved it left of the numeral 8 tent. He pointed at each dot on the tent, counting, “1, 2, 3, 4, 5, 6.” Myra smiled as she turned to Tamara and exclaimed, “Oh! 6 comes before 8!”

Tamara’s actions supported Chris’ and Myra’s problem-solving interactions. Myra and Chris were accountable to one another and classroom norms for respectful participation while collaborating. Myra observed Chris’ relocation of her tent, and Chris listened to Myra as she interpreted his actions.

In Tamara’s reflection of the professional development course, she described the important role she could play in supporting student sense making about mixing representations in number sequences in her Clothesline Math routine. “My next step will be to build a repertoire of questions that would help students without giving them answers.” She reflected that these questions would encourage productive struggle for students (Table 3).

Day 2. The next day, Tamara’s students returned to their clothesline to explore number neighbors as benchmarks when they were counting from 1–20. Tamara removed tents 8 and 9 while leaving 4, 6, 18, 19, and 20. Tamara handed out tents that were “neighbors” to existing numbers on the clothesline and asked students to think where their numbers would go. Kelvin came to the clothesline holding his 15 tent. He paused and softly began counting up from 1 (Figure 6).

Figure 4 Myra’s Placement of the “6” Number Tent



Figure 5 Chris Suggests a Different Place for the “6” Number Tent

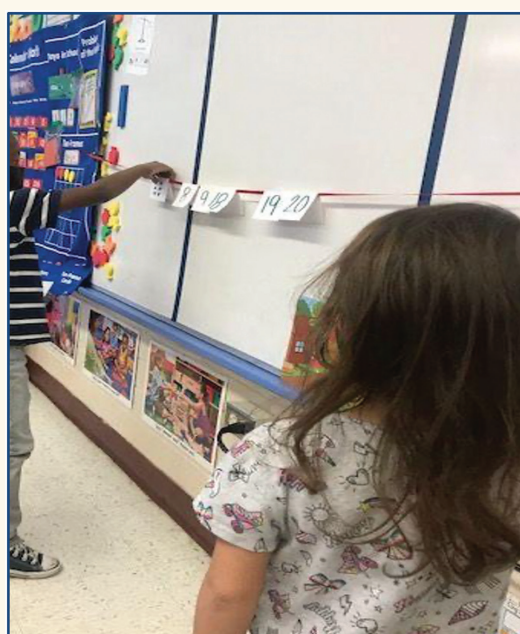


Table 3 Potential Questions for Facilitating Clothesline Math

Focus for Student Thinking	Example Questions
Create an entry point to number placement.	“What is your number?” “What numbers do you see on the clothesline?”
Draw attention to the placement of other numbers.	“Are you standing near where you would like to place your number?” “How do you know where to place your number?” “Is your number more than _____?” “Is your number less than _____?”
Elicit understanding of number relationships.	“What is the starting number on the clothesline? What is the ending number on the clothesline? Is your number close to one of those numbers?” “Is your number greater than 5? 10?” “I see that you are holding the number _____. What is one more than your number? One less than your number?” “Can you find a number on the clothesline to help you locate where your number goes?” “What would be the middle (center) number on the clothesline?” “Is your number more than the middle number?” “Is your number less than the middle number?”

Figure 6 Kelvin Places the “15” Number Tent

After he counted, “18, 19, 20,” he pointed to 18 and asked, “Can I move them?” Tamara nodded, affirming Kelvin’s authority to shift the mathematical conversation in a new direction. Kelvin reached up to slide 18 closer to 19. “18 and 19 should be next to each other. 15 goes here,” he spoke as he placed his 15 tent to the left of 18.

Tamara asked Kelvin to share why he moved the 18 tent. He responded, “18, then 19. They’re next to each other.” Tamara turned to students on the carpet, holding everyone accountable for reasoning about Kelvin’s placement, and asked, “How did Kelvin know where to place the 15?” Students responded:

- “18 and 19 are next-door neighbors and need to stay close!”
- “Kelvin left space between 15 and 18.”
- “There are numbers between 15 and 18.”
- “15 is less than 18!”

Tamara further problematized counting and sequencing for students because of Kelvin’s ideas and decision making. Clothesline Math and its accompanying conversations became resources encouraging students to move beyond rote counting. Students had opportunities to think flexibly about spatial relationships between numbers as a precursor for more advanced reasoning with number lines.

Diane's Grade 3 Clothesline Math Routine

Diane implemented Clothesline Math with Grade 3 students in a series of increasingly rigorous approaches to *problematizing* counting, place value, and mathematical operations. Across successive activities, she gradually increased student *authority* and *accountability*. In the first activity, students sequenced multiples of 10 numerical representations on tents Diane had created. In the second activity, students evaluated addition expressions for multiples of 100 on tents before sequencing tents on the clothesline. In the third activity, students created their own tents to sequence a variety of representations of multiples of four on a shared bulletin board *resource*.

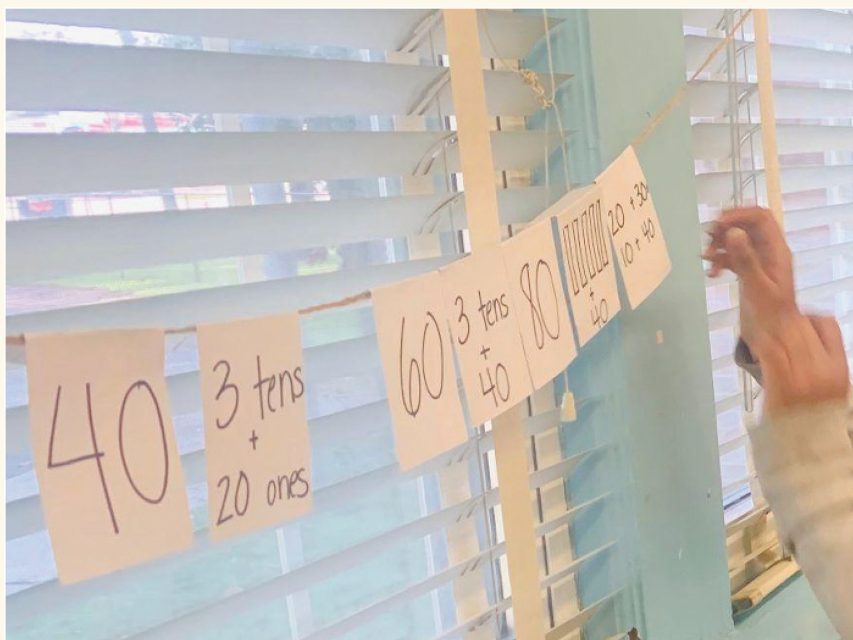
Activity 1: Whole group sequencing of place value representations. Diane engaged student teams in a whole group activity to sequence place value representations of multiples of 10 up to 100. By introducing classroom norms for Clothesline Math in a review of prerequisite place value and counting skills, Diane laid the foundation for using this routine for multiplication. She problematized skip counting by 10s to 100 by creating tents using a variety of numerals, words, arithmetic expressions, and Base-Ten block

drawings (Figure 7). Some numbers were represented as decompositions of place value position amounts. Tents used for this activity were as follows: 40, 3 tens + 20 ones, 60, 3 tens + 40, 80, (five base-ten block rods) + 40, 20 + 30 + 10 + 40.

Diane started by placing a “10 + 10” benchmark tent on a clothesline strung across the window. Students faced several challenges interpreting varied representations. Lucy's group was assigned the “3 tens + 20 ones” tent. In their initial reasoning about this representation of 50, they ignored place value words and attended only to numbers. Lucy incorrectly said, “We have 23,” as she placed their tent to the left of 40. Diane responded with a series of prompts to give Lucy's group authority and accountability for correcting their mistakes (Figure 8).

Another group experienced challenges correctly composing numbers represented with Base-Ten rods and units. Kyle's group was assigned the “5 rods + 40” tent. Diane listened as group members struggled to correctly interpret the mix of Base-Ten rods and units. One student saw five rods as 5, while another saw five rods as five tens or 50. A different student got stuck transitioning from 50 to adding 40. Diane scaffolded student thinking by suggesting they use fingers to keep track of skip counts. Finally, group members

Figure 7 Skip Counting by 10s With Clothesline Math



skip-counted by tens together from 50 to 90: “50, 60, 70, 80, 90.”

Diane was excited about how her integration of place value concepts and skip counting by 10s elicited productive mistake making. In her professional development Google Slideshow template, Diane reflected, “Adjustments were needed throughout the activity, but students eventually got the hang of things. This activity sparked conversations among individuals and groups and encouraged students to embrace mistakes.”

Activity 2: Small group sequencing of addition expressions. Diane created a center where students could collaborate with peers to practice multi-digit addition and represent multiples of 100 using Clothesline Math (Figure 9). To problematize skip counting, she provided resources (e.g., tents with addition expressions having two 2-digit and two 3-digit numbers, Base-Ten blocks, white boards, and markers). Expressions on tents were as follows: $50 + 50$, $180 + 20$, $225 + 75$, $200 + 200$, $320 + 180$, $276 + 324$, $372 + 228$, $229 + 571$, and $535 + 365$. Diane gave students authority to create physical models of addition operations using Base-Ten blocks and whiteboards before sequencing addition expression tents on the clothesline.

Liam used Base-Ten blocks to represent $372 + 228$ (Figure 9). He used his solution of 600 to help Marissa place the “ $372 + 228$ ” tent to the right of the “ $200 + 200$ ” tent because, as Liam said, “500 is one hundred larger than 400 and should come *after* it.” Students were accountable to one another because modeling

addition instead of relying only on mental math made sequencing expressions more accessible to students (Boote & Boote, 2018).

Activity 3: Sharing and sequencing representations of multiples of four. Diane invited students to help create a bulletin board for the school’s Grade 3 hallway (Figure 10). She problematized this authentic task by asking students to create multiples of four representations using repeated addition expressions, equal group images, arrays, and multiplication expressions. Diane gave students authority to create their own tents. After designing tents, students sequenced them on their “Fall into Math” bulletin board. As students used clothespins to place their multiples of four representations, the bulletin board became a valuable resource for sense making. It also became an original artifact on which they could make their mathematical creativity visible to students, teachers, and parents.

CONCLUSION

Clothesline Math is an instructional routine that provides students with greater access to high-cognitive-demand mathematical tasks. Consistent with productive disciplinary engagement principles (Engle, 2012; Engle & Conant, 2002), teachers can problematize mathematics to encourage students to participate more fully in conversations about numerical concepts. Every student takes an active role in the collaborative construction of mathematical knowledge (Agarwal & Sengupta-Irving,

Figure 8 Question Prompts to Guide Student Thinking

<div data-bbox="373 1503 606 1776" data-label="Equation-Block"> $\begin{array}{c} 3 \text{ tens} \\ + \\ 20 \text{ ones} \end{array}$ </div>	<div data-bbox="671 1478 855 1503" data-label="Section-Header"> <p>Diane’s Prompts:</p> </div> <div data-bbox="671 1507 1129 1646" data-label="List-Group"> <ul style="list-style-type: none"> “Lucy, please read your tent to us.” “What are your numbers?” “How many tens do you have?” “How many ones do you have?” “What is your action on your number values?” </div> <div data-bbox="671 1675 943 1701" data-label="Section-Header"> <p>Lucy’s Revised Thinking:</p> </div> <div data-bbox="671 1705 1260 1843" data-label="Text"> <p>“Our numbers are three tens and twenty ones, and we are adding them together. We started with the three tens (another group member says 30), then we skip-counted up the 20 ones which is two tens (group skip counts from 30 to 40 to 50). Our answer is 50, not 23.”</p> </div>
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2018), believing they are safe questioning other's ideas and having their ideas questioned as they deepen mathematical understandings.

Tamara's Pre-K examples illustrate how questions and conversations among teachers and students during Clothesline Math create an environment of authority

and accountability for young mathematics learners. Opportunities to engage in conceptual subitizing through Clothesline Math in Pre-K begin a learning trajectory supporting place value development, model-based thinking about addition, and conceptual approaches to multiplication in later grades. Diane's

Figure 9 Modeling Multi-Digit Addition Expressions in a Clothesline Math Center

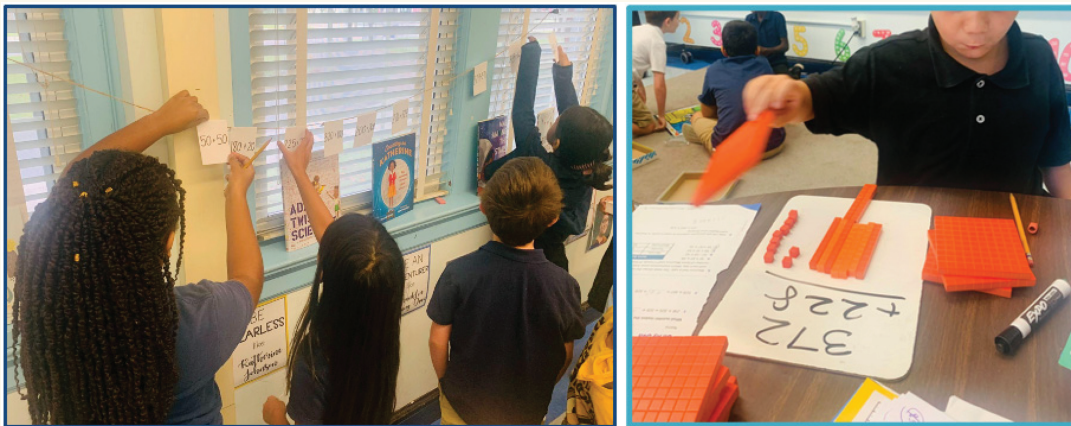


Figure 10 Representing Multiples of Four on a School Bulletin Board



Grade 3 examples illustrate how Clothesline Math can connect prerequisite concepts to new concepts in the context of whole-group discussions, small-group center activities, and a schoolwide authentic task.

By sequencing tents on life-size clotheslines, early childhood students productively engaged in the discipline

of mathematics with their teachers and peers. Teachers reported seeing an increase in students' enthusiasm and willingness to discuss their sense making during routines. We encourage other teachers to build upon these examples to create and share their Clothesline Math routines with the NCTM community. [—](#)

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